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Common Fixed Points for Commuting and Weakly Compatible Self-maps on Digital Metric Spaces

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Abstract: In this paper, we introduce the notions of commutating, compatibility and weakly compatible mappings on digital metric spaces. Using this concept we prove some common fixed point theorems for a pair of self-maps on a digital metric space. We also give an example of a pair of self-maps which is weakly compatible but not compatible and give another example in support of our main result.

Keywords: Digital Image, digital Metric Space, Adjacency relation, Commuting mappings, Compatibility mappings, Weakly Compatible Mappings, Coincidence Point. **2010 MSC:** 47H10, 54E35, 68U10

1. INTRODUCTION

Fixed point theory plays an important role in functional analysis, and it has wider applications in differential and integral equations. Fixed point theory, broadly speaking, demonstrates the existence, uniqueness and construction of fixed points of a function or a family of functions.
The concept of a metric space was introduced by M. Ferchet [15] in 1906.
Fixed point theory has a good beginning from Banach contraction principle of Banach [1] (1922) with complete metric space as back ground. Many authors studied, extended, generalized and improved Banach fixed point theorem in many ways.
In 1976, G. Jungck [24] introduced commuting maps in a complete metric space. This result was generalized and extended for commuting mappings in various ways with several contractive types by many authors [7, 8, 9, 13, 22, 31, 36, 37].
Furthermore, B. E. Rhoades and S. Sessa [33] and S. Sessa [38] extended the result of K. M. Das and K. V. Naik [8] using the notion of generalized commuting mappings called weakly commuting mappings [14, 39, 40].
In 1986, G. Jungck [25] introduced more generalized concept of commutativity, called compatibility. This concept is more general than that of the weak commutativity due to S. Sessa [38]. In 1988, G. Jungck [23] proved some common fixed point theorems for weakly compatible mapping under several contractive conditions.

G. Jungck [27] defined a pair of self-mappings to be weakly compatible if they commute at their coincidence points. Various authors have introduced coincidence point results for various classes of mappings on metric spaces. For more details of coincidence points theory and related results see [26, 28, 32].
Many Authors, used this concept and proved common fixed point theorems on generalized metric spaces like Menger space, d - complete topological space, F – complete metric space, G – metric space, Fuzzy metric space, Cone Metric Spaces, etc.
Now we introduce this concept of digital metric spaces. Digital metric space is one of the generalizations of metric space and digital topology.
Digital topology is a developing area of general topology and functional analysis which studies feature of 2D and 3D digital image. Digital topology is the study of the topological properties of images arrays. A. Rosenfeld [34, 35] was the first to consider digital topology as a tool to study digital images. Kong [29], then introduced the digital fundamental group of a discrete object. The digital version of the topological concept was given by L. Boxer [2, 3, 4].
A. Rosenfeld [35] first studied the almost fixed point property of digital images. Ege and Karaca [11, 12] gave relative and reduced Lefschetz fixed point theorem for digital images. They also calculated the degree of antipodal map for the sphere like digital images using fixed point properties. Ege and Karaca [10] defined a digital metric space and proved the famous Banach Contraction Principle for digital images. But this paper has many slips and was refined and corrected by S. E. Han [21].
Based on these concepts K. Sridevi, M.V.R. Kameswari and D.M.K. Kiran [41] introduced $\varphi -$ contractions and $\varphi -$ contractive mappings on digital metric spaces. They proved an important Lemma and used it to prove the existence and uniqueness of fixed point theorems in digital metric spaces.

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In this paper we introduce commutativity, compatibility and weak compatibility mappings on digital metric spaces and prove some common fixed point theorems on digital metric spaces.

II. PRELIMINARIES

Let X be a subset of $\mathbb{Z}^n$ for a positive integer n where $\mathbb{Z}^n$ is the set of lattice points in the n − dimensional Euclidean Space and ℓ represents an adjacency relation for the members of X. A digital image consists of (X, ℓ).

2.1 Definition (Boxer [3]): Let ℓ, n be positive integers, $1 \le \ell \le n$ and p, q be two distinct points

$$p = (p_1, p_2, \dots, p_n), \quad q = (q_1, q_2, \dots, q_n) \in \mathbb{Z}^n \quad --- (2.1.1)$$

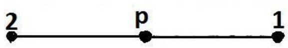
p and q are $\ell -$ adjacent if there are at most ℓ indices i such that $|p_i - q_i| = 1$ and for all other indices j such that $|p_j - q_j| \ne 1, p_j = q_j$.

The following statements can be obtained from Definition 2.1

For a given $p \in \mathbb{Z}^n$, the number of points $q \in \mathbb{Z}^n$ which are $\ell -$ adjacenct to p is denoted by $k(\ell, n)$. It may be noted that $k(\ell, n)$ is independent of p. In practice we write $k = k(\ell, n)$.

1. If $p \in \mathbb{Z}$ (i.e., $n = 1$) then ℓ can take only one value $\ell = 1$. In this case, $k(1,1) = 2$, since $p - 1$ and $p + 1$ are the only points $1 -$ adjacent to p in $\mathbb{Z}$.

Thus, $k = k(1,1) = 2$ and q is $1 -$ adjacent to p if and only if $|p - q| = 1$.



1 - adjacency

2. If $p \in \mathbb{Z}^2$ (i.e., $n = 2$) then ℓ can take values $\ell = 1, 2$.

When $\ell = 2$, the points $2 -$ adjacent to $p = (p_1, p_2)$ are

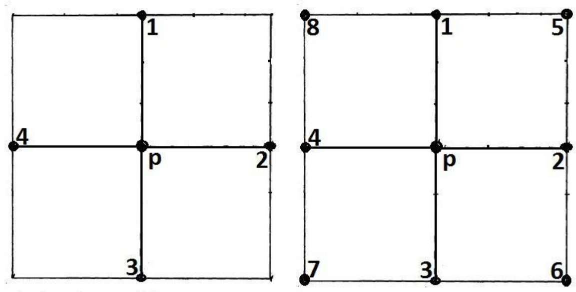
$$(p_1 \pm 1, p_2), (p_1, p_2 \pm 1), (p_1 \pm 1, p_2 \pm 1).$$

Thus, the number of points $2 -$ adjacent to p is 8, so that $k = k(2,2) = 8$. (fig: (b))

When $\ell = 1$, the points $1 -$ adjacent to $p = (p_1, p_2)$ are

$$(p_1 \pm 1, p_2), (p_1, p_2 \pm 1).$$

Thus, the number of points $1 -$ adjacent to p is 4, so that $k = k(1,2) = 4$. (fig: (a))



(a) 1 - adjacency **(b) 2 - adjacency**

3. If $p \in \mathbb{Z}^3$ $(i.e., n = 3)$ then ℓ can take values $\ell = 1, 2, 3$.

When $\ell = 3$, the points $3 - adjacent\ to\ p = (p_1, p_2, p_3)$ are

$$(p_1 \pm 1, p_2, p_3), (p_1, p_2 \pm 1, p_3), (p_1, p_2, p_3 \pm 1), (p_1 \pm 1, p_2 \pm 1, p_3),$$
$$(p_1 \pm 1, p_2, p_3 \pm 1), (p_1, p_2 \pm 1, p_3 \pm 1), (p_1 \pm 1, p_2 \pm 1, p_3 \pm 1).$$

Thus, the number of points $3 - adjacent\ to\ p$ is 26, so that $k = k(3,3) = 26$. (fig: (c))

When $\ell = 2$, the points $2 - adjacent\ to\ p = (p_1, p_2, p_3)$ are

$$(p_1 \pm 1, p_2, p_3), (p_1, p_2 \pm 1, p_3), (p_1, p_2, p_3 \pm 1), (p_1 \pm 1, p_2 \pm 1, p_3),$$
$$(p_1 \pm 1, p_2, p_3 \pm 1), (p_1, p_2 \pm 1, p_3 \pm 1).$$

Thus, the number of points $2 - adjacent\ to\ p$ is 18, so that $k = k(2,3) = 18$.(fig (b))

When $\ell = 1$, the points $1 - adjacent\ to\ p = (p_1, p_2, p_3)$ are

$$(p_1 \pm 1, p_2, p_3), (p_1, p_2 \pm 1, p_3), (p_1, p_2, p_3 \pm 1).$$

Thus, the number of points $1 - adjacent\ to\ p$ is 6, so that $k = k(1,3) = 6$. (fig: (a))

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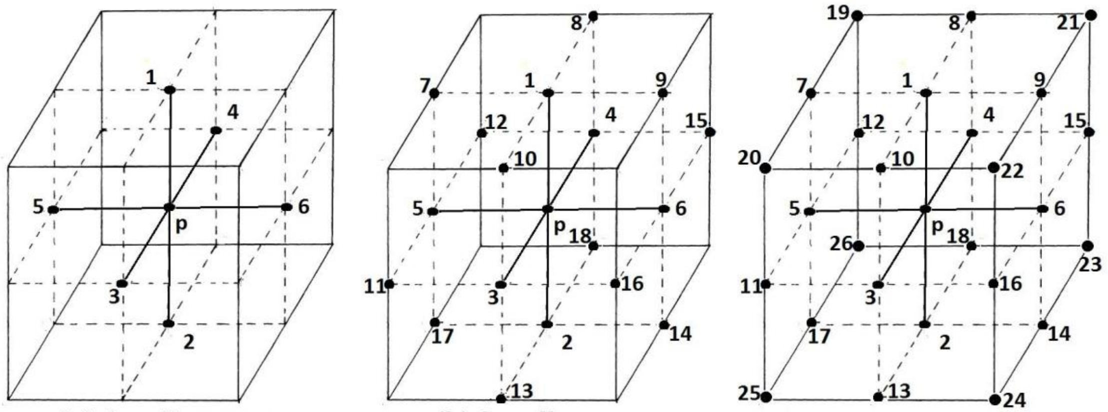

(a) 1 - adjacency (b) 2 - adjacency (c) 3 - adjacency

In general to study nD digital image, if $1 \leq \ell \leq n$ then $k = k(\ell, n)$ is given by the following formula [18] (see also [19, 20]).

$$k(\ell, n) = \sum_{i=n-\ell}^{n-1} 2^{n-i} C_i^n \; --- (2.1.2)$$

where $C_i^n = \frac{n!}{(n-i)!i!}$

Suppose X is a non-empty subset of $\mathbb{Z}^n, 1 \leq \ell \leq n,\ k = k(\ell, n)$. Then (X, ℓ) is called a digital image with $\ell - adjacency$ (Rosenfeld [34]). We also say that (X, ℓ) is called nD digital image [35, 16, 17]. Suppose $p \in \mathbb{Z}^n$ and $1 \leq \ell \leq n$.

Then the digital $\ell - neighborhood$ of p in $\mathbb{Z}^n$(See [34]) is the set

$$N_\ell(p) = \{q | q \text{ is } \ell - adjacent \text{ to } p\}$$

If q is $\ell - adjacent$ to p then we say that p and q are $\ell - neighbours$.

Further, we write (See [10])

$$N_\ell^*(p) = N_\ell(p) \cup \{p\}$$

Suppose $p, q \in \mathbb{Z}$and $p \leq q$. Then the digital interval [30] is defined as

$$[p, q]_{\mathbb{Z}} = \{r \in \mathbb{Z} | \, p \leq r \leq q\}$$

A digital image $X \subset \mathbb{Z}^n$ is said to be $\ell - connected$ [30] if for every two points $u, v \in X$, there is a set $\{u_0, u_1, \ldots, u_r\}$ of points of digital image X such that $u = u_0,\ v = u_r$ and u_i and u_{i+1} are $\ell - neighbours$ for $i = 0, 1, \ldots, r-1$.
Suppose (X, ℓ_0) is a digital image of $\mathbb{Z}^{n_0}$, (Y, ℓ_1) is digital image of $\mathbb{Z}^{n_1}$ and $T : X \longrightarrow Y$ is a function. Then

- T is said to be $(\ell_0, \ell_1) - continuous$ [3], if $\ell_0 - connected$ subsets E of X are mapped into $\ell - connected$ subsets of Y. i.e., E is $\ell_0 - connected$ in X implies $T(E)$ is $\ell_1 - connected$ in Y.
- T is$(\ell_0, \ell_1) - continuous$ if and only if the image of $\ell_0 - adjacent$ points of X are either coincident or $\ell_1 - adjacent$ in Y. i.e., u_0, u_1 are $\ell_0 - adjacent$ points of X then either $T(u_0) = T(u_1)$ or $T(u_0)$ and $T(u_1)$ are $\ell_1 - adjacent$ in Y.
- T is called $(\ell_0, \ell_1) - isomorphism$ [5], if T is $(\ell_0, \ell_1) - continuous$, onto and T^{-1} is $(\ell_1, \ell_0) - continuous$. In this case we write $X \cong_{(\ell_0, \ell_1)} Y$.

2.2 Definition: Suppose $m \in \mathbb{Z}^+$, (X, ℓ) is a digital image in $\mathbb{Z}^n$ and $T : [0, m]_{\mathbb{Z}} \longrightarrow X$ is $(1, \ell) - continuous$.
Suppose $u, v \in \mathbb{Z}$ are such that $T(0) = u$ and $T(m) = v$. Then we say that T is a digital $\ell - path$ [3] from u to v.
Suppose $m \geq 4$, T: $[0, m-1]_{\mathbb{Z}} \longrightarrow X$ is a $\ell - path$ and the sequence $\{T(0), T(1), \ldots, T(m-1)\}$ of images of the $\ell - path$ is such that $T(i)$ and $T(j)$ are $\ell - adjacent$ if and only if $i = j \pm 1 (mod\ m)$. Then we say that T is a simple closed $\ell - curve$ of m points in the digital image (X, ℓ) [6].

2.3 Definition (Han [21]): Let $X \subset \mathbb{Z}^n$, d be the Euclidean metric on $\mathbb{Z}^n$ and (X, d) is a metric space. Suppose (X, ℓ) is a digital image with $\ell - adjacency$. Then (X, d, ℓ) is called a digital metric space.

2.4 Definition (Han [21]): We say that a sequence $\{x_n\}$ of points of the digital metric space (X, d, ℓ) is a Cauchy sequence if there is $M \in \mathbb{N}$ such that
$d(x_n, x_m) < 1$ for all $n, m > M$.

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2.5 Note: Since x_n, x_m are lattice points of $\mathbb{Z}^n$, $\left(d(x_n, x_m)\right)^2$ is a positive integer if $x_n \neq x_m$, and $d(x_n, x_m) = 0$ if $x_n = x_m$.
Consequently, $d(x_n, x_m) < 1 \Longrightarrow d(x_n, x_m) = 0 \Longrightarrow x_n = x_m$.

2.6 Theorem (Han [21]): For a digital metric space (X, d, ℓ), if a sequence $\{x_n\} \subset X \subset \mathbb{Z}^n$ is a Cauchy sequence, there is $M \in \mathbb{N}$ such that for all $n, m > M$, we have $x_n = x_m$.

2.7 Definition (Han [21]): A sequence $\{x_n\}$ of points of a digital metric space (X, d, ℓ) converges to a limit $L \in X$ if for all $\epsilon > 0$, there is $M \in \mathbb{N}$ such that

$$d(x_n, L) < \epsilon \ \ for\ all\ \ n > M.$$

2.8 Proposition (Han [21]): A sequence $\{x_n\}$ of points of a digital metric space (X, d, ℓ) converges to a limit $L \in X$ if there is $M \in \mathbb{N}$ such that
$x_n = L$ for all $n > M$. i.e., $x_n = x_{n+1} = x_{n+2} = \cdots = L$

2.9 Definition (Han [21]): A digital metric space (X, d, ℓ) is complete if any Cauchy sequence $\{x_n\}$ converges to a point L of (X, d, ℓ).

2.10 Theorem (Han [21]): A digital metric space (X, d, ℓ) is complete.

2.11 Definition (Han [21]): Let (X, d, ℓ) be a digital metric space and $T : (X, d, \ell) \longrightarrow (X, d, \ell)$ be a self-map. If there exists $\lambda \in [0, 1)$ such that

$$d(Tx, Ty) \leq \lambda d(x, y) \quad for\ all\ \ x, y \in X,$$

then T is called a contraction map.

2.12 Proposition (Han [21]): Every digital contraction map $T: (X, d, \ell) \longrightarrow (X, d, \ell)$ is $\ell - continuous$ (Digital continuous).

2.13 Lemma [41]: Let $X \subseteq \mathbb{Z}^n$ and (X, d, ℓ) be a digital metric space. Then there does not exist a sequence $\{x_m\}$ of distinct elements in X, such that

$$d(x_{m+1}, x_m) < d(x_m, x_{m-1}) \quad for\ \ m = 1, 2, \ldots - - - - (2.13.1)$$

III. MAIN RESULT

3.1 Definition: Let S and T be two self-maps on a set X. If $Sx = Tx$ for some x in X then x is called coincidence point of $S\ and\ T$.

3.2 Definition: Suppose that (X, d, ℓ) is a digital metric space and $S, T : X \longrightarrow X$ be two self-maps defined on X. Then S and T are said to be commutative if $STx = TSx$ for all $x \in X$.

3.3 Definition: Suppose that (X, d, ℓ) is a digital metric space and $S, T : X \longrightarrow X$ be two self-maps defined on X. Then $S\ and\ T$ are compatible if

$$d(STx, TSx) \leq d(Sx, Tx) \quad for\ all\ x \in X.$$

3.4 Definition: Suppose that (X, d, ℓ) is a digital metric space and $S, T : X \longrightarrow X$ be two self-maps defined on X. Then $S\ and\ T$ are weakly compatible if

$$d(STx, TSx) = d(Sx, Tx)$$

whenever x is a coincidence point of $S\ and\ T$. i.e., $S\ and\ T$ commute at their coincidence point.

We observe that compatibility implies weak compatibility. The following example shows that weak compatibility does not imply compatibility.

3.5 Example: Let $X = \{0, 1, 2, \ldots\}$ and (X, d, ℓ) be the digital metric space where ℓ is $1 - adjacency$. Let $S\ and\ T$ be defined as

$$Sx = \begin{cases} 1, & if\ \ x = 1 \\ x + 1, & if\ x = 2, 3, \ldots \end{cases} \quad \text{and} \quad Tx = x^2$$

$$STx = \begin{cases} 1, & if\ \ x = 1 \\ x^2 + 1, & if\ x = 2, 3, \ldots \end{cases}$$

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$$TSx = \begin{cases} 1, & if\ \ x = 1 \\ (x+1)^2, & if\ x = 2,3,\ldots \end{cases}$$

Hence $$d(STx, TSx) = \begin{cases} 0, & if\ \ x = 1 \\ 2x, & if\ x = 2,3,\ldots \end{cases}$$

$$d(Sx, Tx) = \begin{cases} x^2 - 1, & if\ \ x = 1 \\ x^2 - x - 1, & if\ x = 2,3,\ldots \end{cases}$$

Since $d(ST2, TS2) > d(S2, T2)$ follows that S and $T$are not compatible. But weakly compatible, since 1 is the only coincidence point of S and T and $ST1 = TS1$ and $S1 = T1$.
Now we state and prove our main result.

3.6 Theorem: Suppose (X, d, ℓ) is a digital metric space and $S\ and\ T$ are self maps on X such that $S(X) \subset T(X)$ and

$$d(Sx, Sy) < d(Tx, Ty) \ \ for\ all\ \ x, y \in X \ \ and\ \ x \neq y$$

Then $S\ and\ T$ have unique coincidence point. If further $S\ and\ T$ are weakly compatible then $S\ and\ T$ have unique common fixed point.

Proof: Let $x_0 \in X$. There exists x_1 such that $Sx_0 = Tx_1$.
Then there exists x_2 such that $Sx_1 = Tx_2$.
Inductively, there exists $\{x_n\}$ such that $Sx_n = Tx_{n+1}$ for $n = 0, 1, 2, \ldots$
Suppose, $x_n = x_{n+1}$ for some n.
Then, $Sx_n = Tx_{n+1}$
so that $Sx_{n-1} = Tx_n = Tx_{n+1} = Sx_n$
Therefore, $Sx_n = Tx_n$
and hence, x_n is a coincidence point of S and T.
Hence, we may suppose that, $x_n \neq x_{n+1}$ for $n = 0, 1, 2, \ldots$
Now, $$d(Sx_n, Sx_{n+1}) < d(Tx_n, Tx_{n+1}) = d(Sx_{n-1}, Sx_n) \ \ for\ n = 1, 2, \ldots$$
Therefore, $\{Sx_n\}$ is a finite sequence by **Lemma 2.13.**
Therefore, there exists N such that
$Sx_N = Sx_{N+1} = \cdots$ and
$Tx_{N+1} = Tx_{N+2} = \cdots$
Therefore, $Sx_{N+1} = Sx_N = Tx_{N+1}$
Therefore, x_{N+1} is a coincidence point of S and T.
Suppose x and y are coincidence points of S and T,
so that $Sx = Tx$ and $Sy = Ty$
Suppose, $x \neq y$.
Then, $d(Sx, Sy) < d(Tx, Ty) = d(Sx, Sy)$
a contradiction.
Therefore, $x = y$.
Hence, S and T have unique coincidence point.
Let x be the unique coincidence point of S and Tso that $Sx = Tx$.
Suppose S and T are weakly compatible.
Then, $S(Tx) = T(Sx) = T(Tx)$
Therefore, Tx is a coincidence point of S and T and hence is a common fixed point of S and T.
Hence, S and T have unique common fixed point.

3.7 Corollary: Let S and T be self-maps of a digital metric space (X, d, ℓ). Suppose S and T commute, S(X) ⊂ T(X) and there exits $\alpha \in [0, 1)$ such that

$$d(Sx, Sy) \leq \alpha d(Tx, Ty) \ \ for\ all\ \ x, y \in \mathrm{X} - - - (3.7.1)$$

Then, S and T have unique common fixed point.

Proof: Since S and T commute then they are weakly compatible.
Now, the result follows from **Theorem 3.6**,
since, $\alpha d(Tx, Ty) < d(Tx, Ty)\ \ if\ x \neq y$.

3.8 Corollary: Let S and T be commuting self maps of a digital metric space (X, d, ℓ) and S(X) ⊂ T(X). Suppose there exits $\alpha \in [0, 1)$ and a positive integer k such that

$$d(S^k x, S^k y) \leq \alpha d(Tx, Ty) \ \ for\ all\ \ x, y \in \mathrm{X}$$

Then, S and T have unique common fixed point.

Proof: Since S and T commute, it follows that S^k and T commute. Also $S^k(\mathrm{X}) \subset \mathrm{T(X)}$.
Hence, from the above **Corollary 3.7,** S^k and T have unique common fixed point, say to z.
Then, $S^k(z) = T(z) = z$ so that $S\left(S^k(z)\right) = S\left(T(z)\right) = S(z)$.
Consequently, $T(Sz) = S(Tz) = Sz$

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Hence, $S^k(S(z)) = S(S^k(z)) = S(z)$ so that $S(z)$ is a fixed point of S^k and T.
Consequently, $S(z) = z$. Thus z is a fixed point of S.
Therefore, z is a common fixed point of S and T.
Now, (3.7.1) shows S and T have unique common fixed point.

3.9 Theorem: Let S and T be self maps on a digital metric space (X, d, ℓ) and

$$d(STx, TSy) \leq \lambda d(Tx, Sy) \quad for\ x, y \in X,\ x \neq y.$$

where $0 \leq \lambda < 1$. Then S and T have unique common fixed point.

Proof: Let $x_0 \in X$. Define a sequence $\{x_n\}$ inductively as follows.

$x_1 = Sx_0$
$x_2 = Tx_1$
$x_3 = Sx_2 \ldots$

In general, $x_{2n+1} = Sx_{2n}$ $for\ n = 0, 1, 2, \ldots$ and
$x_{2n} = Tx_{2n-1}$ $for\ n = 1, 2, \ldots$

Consider, $d(STx_1, TSx_0) \leq \lambda d(Tx_1, Sx_0) = \lambda d(x_2, x_1)$

Therefore, $d(x_3, x_2) \leq \lambda d(x_2, x_1)$

In general,

$$\begin{aligned} d(x_{2n+1}, x_{2n}) &= d(Sx_{2n}, Tx_{2n-1}) \\ &= d(STx_{2n-1}, TSx_{2n-2}) \\ &\leq \lambda d(Tx_{2n-1}, Sx_{2n-2}) \\ &\leq \lambda d(x_{2n}, x_{2n-1}) \end{aligned}$$

Therefore, $d(x_{2n+1}, x_{2n}) \leq \lambda d(x_{2n}, x_{2n-1})$ ----- (3.9.1)

$$\begin{aligned} d(x_{2n+2}, x_{2n+1}) &= d(Tx_{2n+1}, Sx_{2n}) \\ &= d(TSx_{2n}, STx_{2n-1}) \\ &\leq \lambda d(Sx_{2n}, Tx_{2n-1}) \\ &\leq \lambda d(x_{2n+1}, x_{2n}) \end{aligned}$$

Therefore, $d(x_{2n+2}, x_{2n+1}) \leq \lambda d(x_{2n+1}, x_{2n})$ ----- (3.9.2)

From (3.9.1) and (3.9.2)

$$d(x_{m+1}, x_m) \leq \lambda d(x_m, x_{m-1}) \qquad for\ m = 1, 2, \ldots$$

Therefore, $\{x_m\}$ is finite sequence by **Lemma 2.13.**
Therefore, $x_{2n} = x_{2n+1} = x_{2n+2} = \cdots$
Therefore, $x_{2n} = Sx_{2n} = Tx_{2n+1} = \cdots$
Therefore, x_{2n} is a fixed point of S.

$$Tx_{2n+1} = x_{2n+2} = x_{2n+1} \ldots$$

Therefore, x_{2n+1} is a fixed point of T.
Therefore, x_{2n} is a fixed point of T.
Therefore, x_{2n} is a common fixed point of S and T.
Now suppose x and y are common fixed points of S and T.
Suppose $x \neq y$. Then

$$\begin{aligned} &d(STx, TSy) \leq \lambda d(Tx, Sy) \\ &d(Sx, Ty) \leq \lambda d(x, y) \\ &d(x, y) \leq \lambda d(x, y) \\ &(1 - \lambda) d(x, y) \leq 0 \end{aligned}$$

Therefore, $d(x, y) = 0 \Longrightarrow x = y$

Therefore, S and T have a unique common fixed point.
Now we give an example in support of our result **(Theorem 3.6)**.

3.10 Example: $X = \{1, 2^1, 2^2, 2^3, \ldots\}$ with $1-adjacency$ and $S, T : X \to X$.

Define $S2^n = \begin{cases} 2^{n-1} & if\ \ n \geq 1 \\ 1 & if\ \ n = 0 \end{cases}$ and $T2^n = 2^n \quad if \quad n = 0,1,2,\ldots$

we observe that $S(X) \subseteq T(X)$ and $d(Sx, Sy) < d(Tx, Ty)$ $\quad for\ all\ \ x, y \in X,\ x \neq y.$

Let $x, y \in X$ and $x = 2^m, y = 2^n$ Then

$$d(Sx, Sy) = \begin{cases} 2^{n-1}(1 - 2n) & if \quad m, n \geq 1 \\ 0 & if \quad m = n = 0 \end{cases}$$

$$d(Tx, Ty) = 2^n(1 - 2^{m-n}) \qquad if \quad m, n = 0,\ 1,\ 2,\ \ldots..$$

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and $S1 = T1$

Therefore, $2^0 = 1$ is the coincidence point of S and T.

Also $S1 = T1 = 1$

Therefore, $2^0 = 1$ is the fixed point of S and T

and $d(ST1, TS1) = d(S1, T1)$

Therefore, S and T are weakly compatible.

Thus all the hypothesis of **Theorem 3.6** satisfied, and

1 is the unique fixed point of S and T.

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